Exam. Code : 103205
Subject Code : 1191

B.A./B.Sc. 5th Semester
MATHEMATICS
Paper—II
(Number Theory)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt ***five*** questions in all, selecting at least ***one*** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) If n is even, prove that $n(n+1)(n+2)$ is divisible by 24. 5

   (b) Establish that, if a and b are odd integers, then $8/(a^2 - b^2)$. 5

2. (a) Find the gcd of 1109 and 4999. Express it in the form $1109x + 4999y$. 5

   (b) Find all solutions in positive integers of the Diophantine equation $172x + 20y = 1000$. 5

SECTION—B

3. (a) If x and y are real numbers, prove that $[x] + [y] \leq [x+y]$, [x] denotes greatest integer functions. 5

   (b) Find the number and sum of divisors of 540. 5

4. (a) If p and 2p+1 both are odd primes and m = 4p then prove that $\phi(m+2) = \phi(m)+2$, ϕ is Euler's phi function. 5

   (b) By using Mobius Inversion formula, find the value of $\phi(n)$. 5

SECTION—C

5. (a) Show if m is an integer then

   $m \equiv 2 \pmod 3$ or $m^2 \equiv m \pmod 6$. 5

   (b) Show that $53^{103} + 103^{53}$ is divisible by 39. 5

6. (a) Solve $17x \equiv 9 \pmod{276}$. 5

   (b) Solve $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$ by Chinese Remainder Theorem. 5

SECTION—D

7. (a) By Euler's theorem show that $a^{560} \equiv 1 \pmod{561}$ if gcd (a, 561) = 1, however 561 is not a prime. 5

   (b) By Fermat's theorem, show that :

   $a^7 \equiv a \pmod{42}$ for all $a \in Z$. 5

8. (a) Using Wilson's theorem, prove that an integer $p > 1$ is a prime number iff $\lfloor p-2 \equiv 1 \pmod p$. 5

   (b) Encrypt the message "RETURN HOME" using Caesar Cipher. 5